

Current dependence of the 'red' boundary of superconducting single photon detectors in the modified hot spot model

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(Dated: August 18, 2014)

We find the relation between the energy of the absorbed photon and the threshold current at which the resistive state appears in the current-carrying superconducting film with the probability about unity. In our calculations we use the modified hot spot model, which assumes different strength of suppression of the superconducting order parameter in the finite area of the film around the place where the photon is absorbed. To find the threshold current we solve the Ginzburg-Landau equation for superconducting order parameter, which automatically includes the current continuity equation and it allows us to consider the back effect of current redistribution near the hot spot on the stability of the superconducting state. We find quantitative agreement with the recent experiments, where we use the single fitting parameter which describes what part of the energy of the photon goes for the local destruction of the superconductivity in the film.

PACS numbers: 74.25.Op, 74.20.De, 73.23.-b

I. INTRODUCTION

In 2001 it was demonstrated that the current-carrying superconducting film biased near its critical current can detect single photon in the visible and near infrared range of electromagnetic radiation (act of detection was seen in the experiment as an appearance of the voltage pulse in the film) [1]. The one of the main characteristics of such a superconducting single photon detector (SSPD) is the position of 'red' boundary, or the largest wavelength of the photon (λ_{max}) which could be detected with probability close to unity. In reality, the part of photons can miss the active element of SSPD - superconducting film, which automatically makes its system detection efficiency smaller than unity. This problem has a technical character and may be solved, for example, by putting a superconducting film in the form of meander to the resonator [2]. To characterize the intrinsic ability of superconducting film to detect the photons it was offered the new characteristic - intrinsic detection efficiency (IDE) [3], which has a meaning of probability of the photon detection after its absorption *by the superconducting film* (from this definition it follows that $IDE \simeq 1$ when $\lambda \leq \lambda_{max}$).

In many experiments it was observed that λ_{max} depends on the transport current (see for example [4–7]). Moreover at fixed λ IDE does not drop suddenly when current becomes smaller some critical (threshold) value but it decreases gradually until it reaches unmeasurably small values. At the moment there are several theoretical models [4, 5, 8, 12] which relates the threshold current I_{thr} with λ_{max} . These models are based on the assumption that the absorbed photon creates the hot spot [4, 5, 8–11] or the hot belt [12] in the superconducting

film (for their comparison see Ref. [7]). In the present work we use a bit different model for the hot spot (see section 2) than the one used in Ref. [8]. To study the situation when $IDE \simeq 1$ we place the hot spot in the center of the film and identify the critical current of such a system as a threshold current. This assumption is based on our recent study [13] where we find that with current increase IDE gradually increases from ~ 0.05 (when only the photons absorbed near the edge of the film provide the instability of superconducting state) up to the unity (when the instability arises also from the photons absorbed in the center of the film). We make our calculations for the films with different width and the hot spots with different size, which correspond to the photons with different energies. We compare our results with recent experiments [6, 7, 14] and find good quantitative agreement with the only assumption that about 10% of the photon energy goes for the local suppression of the superconductivity.

II. MODEL

We consider a two-dimensional superconducting film with finite width and the hot spot is modelled by the local area (in the form of the circle - see insets in Fig. 1(a)) where the quasiparticle distribution function $f(\epsilon)$ is supposed to be far from the equilibrium. We assume that this nonequilibrium is created by the photon and it affects the stability of the superconducting state with transport current. To find the value of the critical current of the film with such a hot spot we numerically solve the Ginzburg-Landau equation for the superconducting order

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parameter $\Delta = |\Delta|e^{i\varphi}$

$$\xi_{GL}^2 \left(\frac{\partial^2 \Delta}{\partial x^2} + \frac{\partial^2 \Delta}{\partial y^2} \right) + \left(1 - \frac{T_{bath}}{T_c} + \Phi_1 - \frac{|\Delta|^2}{\Delta_{GL}^2} \right) \Delta = 0 \quad (1)$$

with the additional term [15–17]

$$\Phi_1 = \int_{|\Delta|}^{\infty} \frac{2(f^0 - f)}{\sqrt{\epsilon^2 - |\Delta|^2}} de, \quad (2)$$

which describes the effect of nonequilibrium distribution function $f(\epsilon) \neq f^0(\epsilon) = 1/(\exp(\epsilon/k_B T_{bath}) + 1)$. In Eq. (1) $\xi_{GL}^2 = \pi \hbar D / 8 k_B T_c$ and $\Delta_{GL}^2 = 8 \pi^2 (k_B T_c)^2 / 7 \zeta(3) \simeq 9.36 (k_B T_c)^2$ are the zero temperature Ginzburg-Landau coherence length and the order parameter correspondingly. Note, that imaginary part of Eq. (1) leads to the current continuity equation $\text{div} j_s = 0$ (j_s is a superconducting current density) which allows us to find the proper distribution of the current density in the film with the hot spot.

It is convenient to write Eq. (1) in dimensionless units (length is scaled in units of $\xi(T_{bath}) = \xi_{GL} / (1 - T_{bath}/T_c)^{1/2}$ and $|\Delta|$ in units of $\Delta_{eq} = \Delta_{GL} (1 - T_{bath}/T_c)^{1/2}$)

$$\frac{\partial^2 \tilde{\Delta}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\Delta}}{\partial \tilde{y}^2} + \left(\alpha - |\tilde{\Delta}|^2 \right) \tilde{\Delta} = 0, \quad (3)$$

where $\alpha = (1 - T_{bath}/T_c + \Phi_1) / (1 - T_{bath}/T_c)$. In Eq. (3) the impact of the absorbed photon on the superconducting properties of the film is described by the single parameter α , which in reality should vary in time and space due to $f(\epsilon, \vec{r}, t)$ (in equilibrium $\alpha = 1$). In our model we use the static approach for the hot spot (with $\alpha(t) = \text{const}$ inside the hot spot). We can do it because of different time scales existing in this problem. Indeed, during initial very short time the hot quasiparticles (appearing after photon absorption - see for example [18]) undergo the downconversion cascade to the energy level just above Δ and the further relaxation process develops much longer. The low energy nonequilibrium quasiparticles diffuse in space (which is relatively slow process due to small group velocity of quasiparticles seating at the energies close to the energy gap), but simultaneously they suppress Δ locally below its equilibrium value. It results in the appearance of quasiparticles with the energy less than Δ_{eq} which already cannot diffuse and they become trapped by the hot spot (see more detailed discussion of this phenomena in Ref.[18]). These quasiparticles can relax to the equilibrium only via electron-phonon scattering with characteristic inelastic electron-phonon relaxation time τ_{e-ph} . Therefore roughly this time governs the final stage of evolution of the hot spot. But the time change of Δ is much faster process and at low temperatures it is proportional to \hbar / Δ_{eq} which is much shorter than τ_{e-ph} . Therefore on the time scale of change of Δ one may consider the quasiparticle distribution function as a static object (at the final stage of its evolution).

In our previous paper [8] in the numerical simulations we used the model with the local temperature of the quasiparticles and solved the heat conductance equation for space and temporal evolution of T_{loc} . This model oversimplifies the real situation because it does not take into account the aforementioned suppressed diffusion of the quasiparticles with $\epsilon \lesssim \Delta_{eq}$ and, besides, it assumes implicitly that at any moment in time the quasiparticles are in the local equilibrium which is rough approximation in the real SSPD where inelastic electron-electron relaxation time is comparable with inelastic electron-phonon relaxation time. But nevertheless it gives *qualitatively* the same results as using here the 'static' hot spot model. It is not surprising, because in the case when $f(\epsilon)$ is described by the Fermi-Dirac function with the local temperature T_{loc} the our control parameter $\alpha(\vec{r}, t) = (1 - T_{loc}(\vec{r}, t)/T_c) / (1 - T_{bath}/T_c)$. Physically it corresponds to the step like distribution of T_{loc} in space which provides qualitatively similar suppression of Δ as it does the Gaussian-like distribution of T_{loc} following from the heat conductance equation (see for example Eq. (13) in Ref. [8]).

Note, that different $f(\epsilon)$ may provide the same value of α , which is consequence of dependence of superconducting order parameter Δ on integral of $f(\epsilon)$ (with some weight function) over the energy. Finding $f(\epsilon)$ needs the solution of the kinetic equation which is very difficult problem [18] and it is beyond the scope of this paper. But to have an insight on the possible values of α let us assume for simplicity that $f(\epsilon)$ is described by the Fermi-Dirac function with the local temperature T_{loc} . Due to diffusion of hot quasiparticles the region where $T_{loc} > T_{bath}$ grows but the most effectively the order parameter is suppressed in the area where $T_{loc} > T_c$ and $\alpha < 0$. At some moment, after the absorption of the photon, the region where $T_{loc} > T_c$ stops to grow and one can use this moment for determination of the effective size of the hot spot.

Based on above consideration we choose $\alpha = 0$ inside the spot in the form of circle with radius R (we also checked what occurs at different values of α). The radius R and the energy of the photon ch/λ are roughly related as

$$\eta \frac{ch}{\lambda} \simeq d \pi R^2 \frac{H_{cm}^2}{8\pi} \quad (4)$$

where $H_{cm} = \Phi_0 / 2 \sqrt{2} \pi \xi \lambda_L$ is the thermodynamic magnetic field, Φ_0 is the magnetic flux quantum, λ_L is the London penetration depth, d is the thickness of the film and $H_{cm}^2 / 8\pi$ is the superconducting condensation energy per unit of volume. Coefficient $0 < \eta < 1$ takes into account that not whole energy of the photon goes for suppression of Δ [4]. For example the large part of the energy of the photon is delivered to the energy of quasiparticles E_q . One can easily find it in the local temperature approximation (when $T_{loc}(\vec{r}) = \text{const}$ inside the circle with

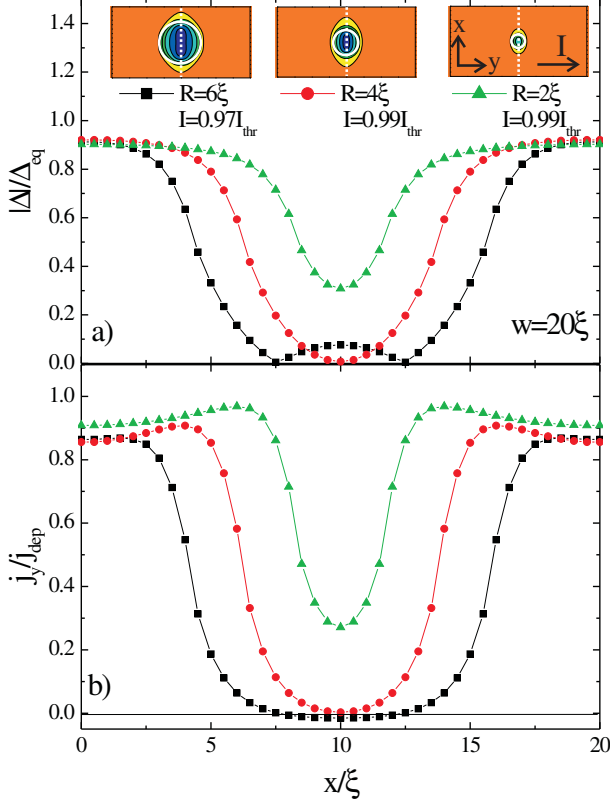


FIG. 1: Distribution of $|\Delta|$ (a) and superconducting current density (b) along the white lines marked in the insets (where we plot contour plot of $|\Delta|$ in the film). The region where we put $\alpha = 0$ is bordered by the white circle in the insets in figure (a) and at all radiuses the transport current is close to the threshold value.

radius R)

$$E_q = d\pi R^2 4N_0 \int_0^\infty \epsilon f(\epsilon) d\epsilon = d\pi R^2 N_0 \frac{\pi^2 k_B^2 T_{loc}^2}{3} \simeq d\pi R^2 N_0 \Delta_0^2 \frac{1.06 T_{loc}^2}{T_c^2} \quad (5)$$

where N_0 is a one-spin density of states of quasiparticles at the Fermi level, $\Delta_0 = 1.76 k_B T_c$ is the superconducting order parameter at zero temperature and we neglect the energy of quasiparticles in equilibrium (which is small in comparison with Eq. (5) when $\Delta_{eq} > k_B T_{bath}$ and $T_{loc} \gg T_{bath}$). One can see that when $T_{loc} = T_c$, E_q is twice larger than the condensation energy (because at low temperatures $H_{cm}^2/8\pi \simeq N_0 \Delta_0^2/2$) and when $T_{loc} = 2T_c$, E_q is larger more than in eight times.

In numerical calculations we consider the film of finite width w and length $L = 4w$ with the hot spot (region where $\alpha < 1$) placed in the center of the film - see insets in Fig. 1a. We also add to the right hand side of Eq. (3)

the term with time derivative $\partial \tilde{\Delta}/\partial t$ (in a way how it was done in Ref. [8]) which allows us to find not only the value of the critical (threshold) current but also to find how the superconducting state becomes destroyed. As an edge effect during the nonstationary stage the normal current j_n (and electrostatic potential) appears in the film and we find them by solving equation $div(j_n + j_s) = 0$.

III. RESULTS

In Fig. 1(a,b) we show distribution of $|\Delta|$ and current density across the hot spot and sidewalks. Note that despite of $\alpha = 0$ the order parameter is finite in the hot spot region (see Fig. 1(a)). It occurs due to proximity effect from the surrounding area and the same effect leads to partial suppression of $|\Delta|$ outside the region where we keep α . Due to the current crowding effect the maximum of current density is located near the 'edge' of the hot spot (compare Fig. 1(a) and 1(b)). The same result follows from the analytical calculations in the London model (see Ref. [8]). This result is opposite to the findings of Ref. [9] (see Fig. 5 there) where authors find the maximal current density at the edge of the film. We checked for different w and R that the current density is maximal at the edge of the film only when the size of the hot spot is comparable with the width of the film (one can see it on the example of the hot spot with diameter $D = 12\xi \sim w = 20\xi$ in Fig. 1(b)).

When the current exceeds some critical value the vortex-antivortex pair appears *inside* the hot spot. It occurs due to large value of the supervelocity ($v_s \simeq j_s/|\Delta|^2$) inside the hot spot [8], which actually affects the superconductivity. For relatively large hot spots (with $R \gtrsim 3\xi$) this pair is unbound at larger current, at which vortex and antivortex move freely in the opposite directions. One can see the signs of the presence of the vortex-antivortex pair from distribution $|\Delta|(x)$ and $j_y(x)$ in Fig. 1(a,b) for the spot with $R = 6\xi$. Dependence $|\Delta|(x)$ has two local minima near the center of the film which correspond to the centers of the vortex and antivortex. From Fig. 1(b) one can see that j_y is negative near the center of the film which is the consequence of the currents flowing around the vortex/antivortex centers and which have opposite direction to the direction of the transport current.

We relate the current, at which the free motion of vortices and antivortices start, with the threshold current at which IDE of SSPD approaches unity (in Ref. [13] we find that when the hot spot located *not* in the center of the film the free vortex motion starts at *lower* currents). In Fig. 2 we present dependence of I_{thr}/I_{dep} on the radius of the spot when $\alpha = 0$. In the same figure we present dependence $I_{thr}(R)$ (solid curves) following from the London model (see Eq. (12) in Ref. [8]). The quantitative difference between these results is not surprising, because in the analytical model of Ref. [8] the gradual variation of $|\Delta|$ at the edge of the hot spot was

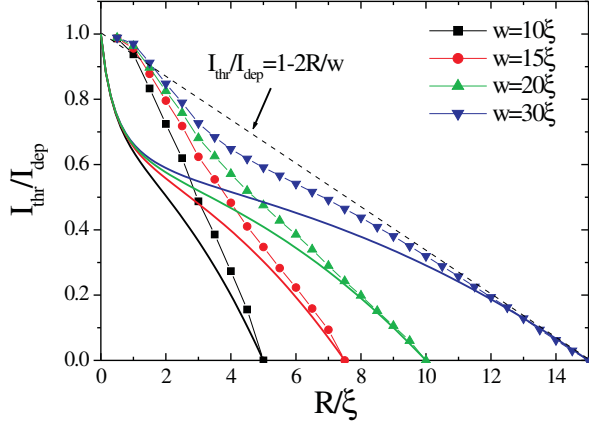


FIG. 2: Threshold current as a function of the radius of the spot ($\alpha = 0$) for films with different widths. Solid curves correspond to Eq. (12) from Ref. [8] with $\gamma = 0$. Dotted line corresponds to the dependence $I_{thr}(R)$ for the film with width $w = 30\xi$ in the model with spatially uniform current distribution in the sidewalks near the hot spot.

neglected and semi-quantitative criteria for unbinding of vortex-antivortex pair was used.

In Fig. 2 we also plot $I_{thr}(R)$ (dashed line) for the film with $w = 30\xi$ which follows from the model with spatially uniform current distribution in the sidewalks [5]. One can see that the current crowding effect leads to smaller value of I_{thr} when $\xi \ll R \ll w$.

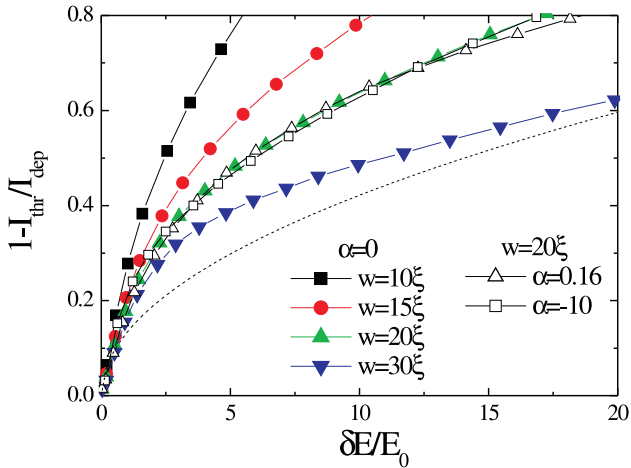


FIG. 3: Dependence of the threshold current on the energy difference between the states of the superconducting film with and without the hot spot (we did not take into account the difference in the energy of quasiparticles).

In Fig. 3 we show the same dependencies but instead of radius of the spot, as one of the coordinate, we use the difference between the free energies of the states of the

film with and without the hot spot

$$\delta E = F_{spot} - F_{nospot} - \frac{\hbar}{2e} I \delta \varphi. \quad (6)$$

where

$$F = -\frac{H_{cm}^2 d}{8\pi} \int \left(\frac{|\Delta|}{\Delta_{eq}} \right)^4 dS, \quad (7)$$

and $\delta \varphi$ is the extra phase difference due to presence of the hot spot. At the derivation of Eq. (7) from the Ginzburg-Landau free energy functional we take into account that $|\Delta|$ is the solution of the Ginzburg-Landau equation and we do not consider difference in the energy of quasiparticles because it strongly depends on explicit dependence $f(\epsilon)$. In Fig. 3 (where the energy is normalized in units of $E_0 = H_{cm}^2 d \xi^2 / 2 = \Phi_0^2 d / 16 \pi^2 \lambda_L^2$) for the film with $w = 20\xi$ we also show the results for the hot spots with different values of α . Surprisingly, the difference is not large in the used coordinates (notice, that for larger α the same value I_{thr} is reached for larger radius of the hot spot, but the energies of these states are turned out to be close to each other).

IV. COMPARISON WITH THE EXPERIMENT

To relate the theoretical results present in Figs. 2,3 with the recent experiments [6, 7, 14] we suppose that only fraction of the photon's energy $\eta \hbar c / \lambda$ goes for destruction of superconductivity inside the hot spot. In Figs. 4,5 we compare our results with the experimental results present in Refs. [6, 7, 14]. In Fig. 4 we use $\xi = 7nm$ and the square resistance $R_{sq} = 400 Ohm$ for all theoretical curves. Depairing current for TaN meander from Ref. [6] is calculated with the Kupriyanov-Lukichev correction [19]. In Ref. [9] this correction was omitted [20] and it led to slightly smaller value of I_{dep} (compare Fig. 4 with the inset in Fig. 9 of Ref. [9]). In Ref. [14] authors did not present parameters of the NbN bridge (R_s or λ_L and T_c) and we take these parameters from Ref. [22] for NbN film with the same thickness as in Ref. [14].

In Figs. 4,5 the only fitting parameter is the coefficient η . It has almost the same value about 0.1 for different TaN meanders. For NbN bridge the best fitting is obtained for $\eta = 0.17$ ((note, that other hot spot models give $\eta \simeq 0.1 - 0.4$ after their comparison with the experiments [4, 6, 9]). We should mention that in Ref. [14] the IDE was fixed at the level ~ 0.25 (this number can be estimated using the experimental results for detection probability p_n in Fig. 3 of Ref. [14]). From Fig. 3 of Ref. [14] one can see that p_n saturates (and IDE $\rightarrow 1$) at larger currents. It means that the experimental points in Fig. 5 should be shifted upwards and in this case the theoretical results would fit the experimental ones practically at the same value of η as for TaN meanders.

There is also another work [21], where the photon detection by the NbN bridge (in the same geometry as in

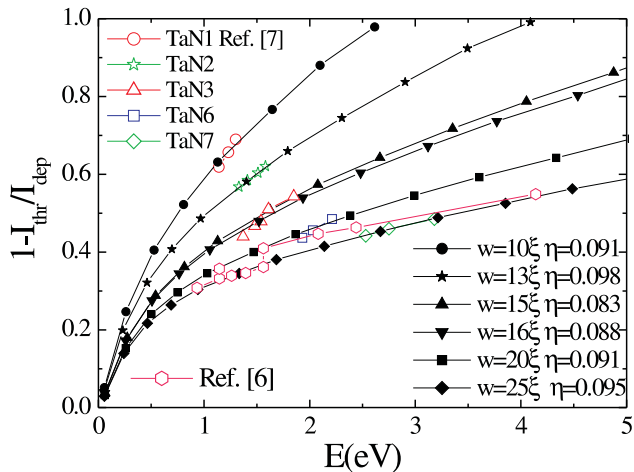


FIG. 4: Dependence of the threshold current, when $IDE \rightarrow 1$, on the energy of the absorbed photon. The experimental results were found for TaN based SSPD in Refs. [6, 7].

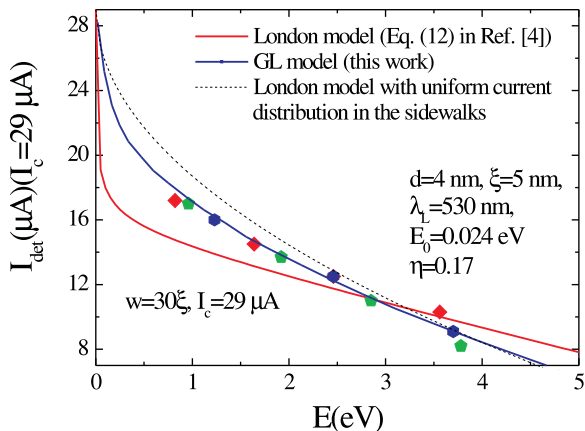


FIG. 5: Symbols - experimental results from Ref. [14] found when $IDE \sim 0.25$. Solid curves - theoretical results calculated in GL and London models when $IDE \sim 1$. Dashed line corresponds to the model with uniform current distribution in the sidewalks near the normal spot.

[14]) was studied. In this work authors fix the detection probability at $p_n = 0.01$ (while in the previous work they used $p_n = 0.1$ [14]) and find linear dependence of I_{thr} on the energy of the absorbed photons, which provides such value of p_n . Assuming the presence of correlation between the studied bridges in Refs. [14, 21] one may suggest that $p_n = 0.01$ corresponds to $IDE \sim 0.025$. In this case we cannot compare directly this experiment with our results because they are obtained when $IDE \sim 1$.

Note, that if one extrapolates the experimental points in Figs. 4,5 (and in Fig. 2 of Ref. [21]) by linear functions then they cross the vertical axis at $I_{thr} \sim 0.7 - 0.8 I_{dep}(I_c)$. This circumstance led to the conclusion in Refs. [9, 21] that the penetration of the vortices via edge energy barrier is important for photon detection process, because in Ref. [12] it was found that

this barrier goes to zero at current smaller when I_{dep} . We should stress here that the energy barrier in Ref. [12] is calculated in the London model and this approach quantitatively fails when the vortex sits on the distance smaller than $\sim 2\xi$ from the edge of the film. Calculations in the framework of the Ginzburg-Landau model [23] demonstrate that the energy barrier for the vortex penetration vanishes *exactly* at $I = I_{dep}$ and only at currents $I < 0.6 I_{dep}$ the London and the Ginzburg-Landau models give the same results (if one takes into account the energy of the vortex core [23]). Note, that if in the film there are some geometric or structural inhomogeneities then the energy barrier for the vortex entry vanishes at the critical current of the film $I_c < I_{dep}$ [23]. In this case $IDE \sim 1$ could be reached only for the photons, whose energy satisfies the inequality $I_{thr}(E) \leq I_c$.

V. CONCLUSION

In this work we exploit the modified hot spot model to find the relation between the energy of the absorbed photon and the threshold current at which the resistive state appears *with probability about unity* in the current-carrying superconducting film. The hot spot is modelled as a finite region in the center of the film where the quasiparticle distribution function is far from the equilibrium and it leads to the partial suppression of the superconducting order parameter. We find that the resistive state starts from the nucleation and unbinding the vortex-antivortex pair *inside* the hot spot. When the energy of the photon goes to zero, the suppression of the order parameter in the hot spot becomes small and the threshold current rapidly approaches the depairing current. Comparison of found results with recent experiments shows good agreement if one assumes that only about 10% of the energy of the photon goes for the local destruction of the superconductivity.

The main differences of our model with the previous hot spot models [4, 5, 9–11] are following: i) we solve the current continuity equation $div j_s = 0$ in the film with the hot spot (which gives us correct distribution of the current density in the superconductor); ii) we take into account the back effect of the current redistribution around the hot spot on the superconducting order parameter. Because we did not solve the kinetic equation and we did not find nonequilibrium distribution function we do not know the actual value of α and we cannot relate quantitatively the energy of the photon with the size of the hot spot and how strong the superconductivity is suppressed inside it. But we find that the relation between the threshold current and the energy needed for suppression of the superconductivity inside and around the hot spot (which does not include the energy of hot quasiparticles and phonons) weakly depends on these parameters. This result encourages us that the found results are rather general and have direct relation to the dependence of the 'red' boundary of SSPD on the trans-

port current.

Acknowledgments

We thank to Alexander Semenov, Gregory Gol'tsman, Alexey Semenov, Jelmer Renema, Michiel de Dood and Martin van Exter for valuable discussion of the found

results. The work was partially supported by the Russian Foundation for Basic Research (project 12-02-00509) and by the Ministry of education and science of the Russian Federation (the agreement of August 27, 2013, N 02.49.21.0003 between The Ministry of education and science of the Russian Federation and Lobachevsky State University of Nizhni Novgorod).

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- [1] G. N. Goltsman, O. Okunev, G. Chulkova, A. Lipatov, A. Semenov, K. Smirnov, B. Voronov, A. Dzardanov, C. Williams, and R. Sobolewski, *Appl. Phys. Lett.* **79**, 705 (2001).
 - [2] F. Marsili, V. B. Verma, J. A. Stern, S. Harrington, A. E. Lita, T. Gerrits, I. Vayshenker, B. Baek, M. D. Shaw, R. P. Mirin, and S. W. Nam, *Nature Phot.* **7**, 210 (2013).
 - [3] M. Hofherr, D. Rall, K. Il'in, M. Siegel, A. Semenov, H.-W. Hübers, and N. A. Gippius, *J. of Appl. Phys.* **108**, 014507 (2010).
 - [4] A. Semenov, A. Engel, H.-W. Hübers, K. Il'in, and M. Siegel, *Eur. Phys. J. B* **47**, 495 (2005).
 - [5] L. Maingault, M. Tarkhov, I. Florya, A. Semenov, R. Espiau de Lamaëstre, P. Cavalier, G. Goltsman, J.-P. Poizat, and J.-C. Villégier, *J. Appl. Phys.* **107**, 116103 (2010).
 - [6] A. Engel, A. Aeschbacher, K. Inderbitzin, A. Schilling, K. Ilin, M. Hofherr, M. Siegel, A. Semenov, and H.-W. Hubers, *Appl. Phys. Lett.* **100**, 062601 (2012).
 - [7] R. Lusche, A. Semenov, K. Ilin, M. Siegel, Y. Korneeva, A. Trifonov, A. Korneev, G. Goltsman, D. Vodolazov, and H.-W. Hübers, *J. Appl. Phys.* **116**, 043906 (2014).
 - [8] A. N. Zotova and D. Yu. Vodolazov, *Phys. Rev. B* **85**, 024509 (2012).
 - [9] A. Engel and A. Schilling, *J. Appl. Phys.* **114**, 214501 (2013).
 - [10] A. D. Semenov, G. N. Gol'tsman, and A. A. Korneev, *Phys. C (Amsterdam)* **351**, 349 (2001).
 - [11] A. Eftekharian, H. Atikian, and A. H. Majedi, *Opt. Express* **21**, 3043 (2013).
 - [12] L.N. Bulaevskii, M.J. Graf, and V.G. Kogan, *Phys. Rev. B* **85**, 014505 (2012).
 - [13] A.N. Zotova and D. Y. Vodolazov, arXiv:1407.3710 [cond-mat.supr-con].
 - [14] J. J. Renema, G. Frucci, Z. Zhou, F. Mattioli, A. Gaggero, R. Leoni, M. J. A. de Dood, A. Fiore, and M. P. van Exter, *Phys. Rev. B* **87**, 174526 (2013).
 - [15] J. J. Renema,¹ G. Frucci,² Z. Zhou,² F. Mattioli,³ A. Gaggero,³ R. Leoni,³ M. J. A. de Dood,¹ A. Fiore,² and M. P. van Exter¹
 - [16] B. I. Ivlev, S. G. Lisitsyn, and G. M. Eliashberg, *J. Low Temp. Phys.* **10**, 449 (1973).
 - [17] A. I. Larkin and Yu. N. Ovchinnikov, *Zh. Eksp. Teor. Fiz.* **68**, 1915 (1975) [*Sov. Phys. JETP* **41**, 960 (1976)].
 - [18] L. Kramer and R.J. Watts-Tobin, *Phys. Rev. Lett.* **40**, 1041 (1978).
 - [19] A. G. Kozorezov, A. F. Volkov, J. K. Wigmore, A. Peacock, A. Poelaert, and R. den Hartog, *Phys. Rev. B* **61**, 11 807 (2000).
 - [20] M. Y. Kupriyanov and V. F. Lukichev, *Sov. J. Low Temp. Phys.* **6**, 210 (1980).
 - [21] A. Engel and A. Semenov, private communication.
 - [22] J. J. Renema, R. Gaudio, Q. Wang, Z. Zhou, A. Gaggero, F. Mattioli, R. Leoni, D. Sahin, M. J. A. de Dood, A. Fiore, and M. P. van Exter, *Phys. Rev. Lett.* **112**, 117604 (2014).
 - [23] A. Kamlapure, M. Mondal, M. Chand, A. Mishra, J. Jesudasan, V. Bagwe, L. Benfatto, V. Tripathi, and P. Raychaudhuri, *Appl. Phys. Lett.* **96**, 072509 (2010).
 - [24] D. Yu. Vodolazov, *Phys. Rev. B* **85**, 174507 (2012).